STABILITY OF STEADY-STATE FLOW OF A LIQUID WITH A HEAVY IMPURITY

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The effect of an impurity in the form of a small quantity of solid particles on the stability of plane-parallel flows of an incompressible gas was studied in [1-4], wherein it was assumed that the particles are homogeneously distributed and do not produce motion of the gas. Below we will study stability of steady-state flow of a liquid with a solid impurity in a vertical plane layer. Liquid motion is produced by settling of the nonuniformly distributed heavy impurity particles. The dependence of flow stability on the character of particle distribution within the layer is demonstrated.

1. We will consider a viscous incompressible liquid, containing an impurity in the form of nondeforming spherical solid particles of radius r and mass m. As in [1-6], we assume the liquid and impurity to be continuous media, interpenetrating and interacting with each other, and neglect interaction between the particles. The volume fraction of particles is assumed to be so low that the Einstein correction to liquid viscosity can be neglected. The density of the particle material ρ_1 is much greater than the density of the carrier medium ρ. The left force acting on the particles is negligibly small, since it is proportional to the ratio $\rho/\rho_1 << 1$. Interaction between the phases as they undergo relative motion follows the Stokes law.

The equations describing the behavior of an incompressible liquid with an impurity of heavy solid particles have the form [7, 8]

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\,\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\Delta\mathbf{u} - \frac{a}{\tau_v}(\mathbf{u}_p - \mathbf{u}) + \mathbf{g}, \tag{1.1}$$

$$\frac{\partial \mathbf{u}_p}{\partial t} + (\mathbf{u}_p\nabla)\,\mathbf{u}_p = \frac{1}{\tau_v}(\mathbf{u}_p - \mathbf{u}) + \mathbf{g},\ \mathrm{div}\,\mathbf{u} = 0,$$

$$\partial\rho_p/\partial t + \mathrm{div}\,\rho_p\mathbf{u}_p = 0,\ \rho_p = mN,\ \tau_v = m/(6\pi r\rho\nu),\ a = \rho_p/\rho,$$

where u is the liquid velocity; p, its pressure; ν, kinematic viscosity; quantities with the subscript p refer to the particle cloud; N, number of particles per unit volume; τ_v, time over the course of which the particle velocity decreases relative to that of the liquid by a factor of e times as compared to its original value; and **g**, acceleration of gravity.

Let the liquid with impurity be located in a plane layer formed by two infinite vertical parallel planes $x = \pm h$. The particles are distributed across the layer symmetrically relative to the vertical z axis (Fig. 1) according to a law

$$N(\alpha, x) = \frac{4\,\mathrm{ch}\,\alpha\,\mathrm{ch}\,\frac{\alpha x}{h} - \mathrm{ch}\,\frac{2\alpha x}{h} - \mathrm{ch}\,2\alpha - 2}{4\,\mathrm{ch}\,\alpha - \mathrm{ch}\,2\alpha - 3}, \tag{1.2}$$

where α is a coefficient defining the impurity concentration near the boundary of the layer (in Fig. 1, $\alpha_1 = 1$, $\alpha_2 = 6$, $\alpha_3 = 20$). Equation (1.2) describes well the distribution of settling particles in a vertical channel observed experimentally in [8].

The settling particles, nonuniformly distributed across the channel, interact with the liquid and set it in motion. We find the steady-state distribution of liquid and particle velocities from Eq. (1.1) with the assumption that trajectories of both liquid and solid particles are straight lines parallel to the z axis, closing at infinity above and below:

$$\frac{1}{\rho}\frac{dp_0}{dz} = \nu\frac{d^2u_0}{dx^2} - \frac{a}{\tau_v}(u_{p0} - u_0) - g,\ \frac{1}{\tau_v}(u_{p0} - u_0) = g. \tag{1.3}$$

Here u_0 and u_{p0} are the vertical velocity components and the subscript 0 indicates the steady-state solution of Eq. (1.1).

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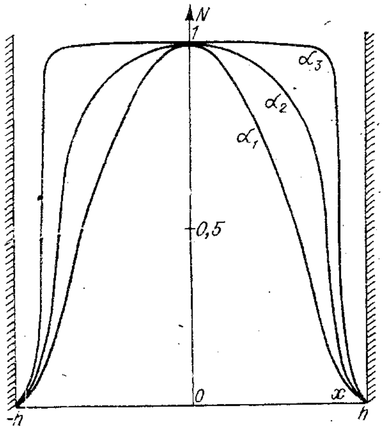

Fig. 1

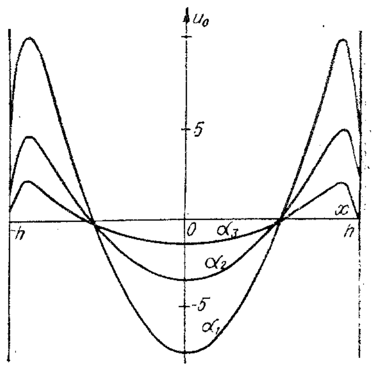

Fig. 2

The boundary conditions and closed flow condition are expressed by

$$u_0(\pm h) = 0, \quad \int_{-h}^{h} u_0 dx = 0. \tag{1.4}$$

Solving the problem of Eqs. (1.3), (1.4), we obtain the steady-state distributions of liquid and particle cloud velocities over the layer section

$$u_0 = \frac{gh^2}{\nu} B_1 \left[\frac{1}{\alpha^2} \left(4 \operatorname{ch} \alpha \operatorname{ch} \frac{\alpha x}{h} - \frac{1}{4} \operatorname{ch} \frac{2\alpha x}{h} \right) + B_2 \frac{x^2}{h^2} - B_3 \right], \tag{1.5}$$

$$u_{p0} = u_0 - g\tau_v, \quad \nabla p_0 = \text{const},$$

$$B_1 = \frac{m}{\rho (4 \operatorname{ch} \alpha - \operatorname{ch} 2\alpha - 3)}, \quad B_2 = \frac{3}{4\alpha^2} \left(\frac{15}{4\alpha} \operatorname{sh} 2\alpha - \frac{7}{2} \operatorname{ch} 2\alpha - 4 \right),$$

$$B_3 = \frac{45}{16\alpha^3} \operatorname{sh} 2\alpha - \frac{7}{8\alpha^2} \operatorname{ch} 2\alpha - \frac{1}{\alpha^2}.$$

As is evident from Eq. (1.5), under the action of the settling particles within the layer a liquid motion is established with two ascending and one descending flow, symmetric about the z axis (Fig. 2, where $\alpha_1 = 21$, $\alpha_2 = 31$, $\alpha_3 = 50$). The intensity of the motion decreases with increase in α (as $\alpha \to \infty$, $u_0 \to 0$).

2. We will study the stability of the steady-state liquid flow of Eq. (1.5), produced by settling of the nonuniformly distributed impurity particles. To do this we impose upon the steady-state velocity fields u_0, u_{p0}, pressure p_0, and number of particles per unit volume N_0, the small perturbations u, u_p, p, N.

We write the equations for the perturbations in dimensionless form, using the following units to dedimensionalize: for distance, h; time, h^2/ν; velocity, ν/h; pressure $\rho\nu^2/h^2$. Linearizing with respect to the perturbations, we obtain from Eq. (1.1)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}_0 \nabla) \mathbf{u} + (\mathbf{u} \nabla) \mathbf{u}_0 = - \nabla p + \Delta \mathbf{u} - \frac{a_0}{\tau_v} (\mathbf{u}_p - \mathbf{u}) + \mathrm{Ga}\, a \boldsymbol{\gamma}, \tag{2.1}$$

$$\frac{\partial \mathbf{u}_p}{\partial t} + (\mathbf{u}_{p0} \nabla) \mathbf{u}_p + (\mathbf{u}_p \nabla) \mathbf{u}_{p0} = \frac{1}{\tau_v} (\mathbf{u}_p - \mathbf{u}),$$

$$\operatorname{div} \mathbf{u} = 0, \quad \frac{\partial N}{\partial t} + \operatorname{div} [N_0 \mathbf{u}_p + N \mathbf{u}_{p0}] = 0;$$

$$u_0 = \mathrm{Ga} B_1 \left[\frac{1}{\alpha^2} \left(4 \operatorname{ch} \alpha \operatorname{ch} \alpha x - \frac{1}{4} \operatorname{ch} 2\alpha x \right) + B_2 x^2 - B_3 \right], \tag{2.2}$$

$$u_{p0} = u_0 - u_s, \quad \mathbf{u}_s = - \mathrm{Ga}\, \tau_v \boldsymbol{\gamma},$$

$$\tau_v = \frac{2}{9} r^2 \frac{\rho_1}{\rho}, \quad a = \frac{mN}{\rho}, \quad a_0 = \frac{mN_0}{\rho},$$

$$\mathrm{Ga} = \frac{gh^3}{\nu^2}, \quad N_0 = \frac{4 \operatorname{ch} \alpha \operatorname{ch} \alpha x - \operatorname{ch} 2\alpha x - \operatorname{ch} 2\alpha - 2}{4 \operatorname{ch} \alpha - \operatorname{ch} 2\alpha - 3},$$

where u_s is the particle settling velocity; Ga is the Galileo number; τ_v is the dimensionless relaxation time; γ is a unit vector directed vertically upward.

For a liquid with impurity [6], as for a pure liquid [9, 10], it has been demonstrated that the problem of stability with respect to spatial perturbations reduces to the problem of stability with respect to planar perturbations. In the case under consideration planar perturbations are more dangerous, i.e., they correspond to lower Galileo numbers, so that in studying stability it is sufficient to limit ourselves to the study of planar normal perturbations:

$$\begin{gathered}\mathbf{u}_p(x, z, t) = \mathbf{v}_p(x) \exp\, [ik(z - ct)],\\ N(x, z, t) = n(x) \exp\, [ik(z - ct)],\\ \psi(x, z, t) = \varphi(x) \exp\, [ik(z - ct)],\\ u_x = -\partial\psi/\partial z,\ u_z = \partial\psi/\partial x.\end{gathered} \tag{2.3}$$

Here ψ is the flow function; φ, $\mathbf{v}_p$, n are the perturbation amplitudes; k is the real wave number; $c = c_r + ic_i$ is the complex phase velocity of the perturbations (c_r is the phase velocity, c_i is the decrement).

Substituting Eq. (2.3) in Eq. (2.1), we obtain the amplitude equation (with the prime denoting differentiation with respect to the coordinate x)

$$(\varphi^{\mathrm{IV}} - 2k^2\varphi'' + k^4\varphi) + ik(\varphi'' - k^2\varphi)\left(c - u_0 + \frac{a_0}{ik\tau_v}\right) + iku_0''\varphi = \frac{a_0}{\tau_v}(v_{pz}' - ikv_{px}) + \frac{a_0'}{\tau_v}(v_{pz} - \varphi') + \mathrm{Ga}\, n', \tag{2.4}$$

$$v_{px} = \frac{ik\varphi}{ik\tau_v(u_{p0} - c) - 1},\quad v_{pz} = \frac{-\varphi' + u_{p0}'\tau_v v_{px}}{ik\tau_v(u_{p0} - c) - 1},$$

$$n = -\frac{ikv_{pz}N_0 + N_0'v_{px} + N_0 v_{px}'}{ik(u_{p0} - c)},$$

with boundary conditions

$$\varphi(\pm 1) = \varphi'(\pm 1) = 0. \tag{2.5}$$

The stability boundary for flow of the liquid with impurity Eq. (2.2) is determined by the condition $c_i = 0$. The complex phase velocity c depends on the problem parameters Ga, k, α, τ_v. To solve the boundary problem Eqs. (2.4) and (2.5), i.e., to determine the stability limits of the flow under consideration and calculate the decrement spectrum, we use the Runge–Kutta method of step-by-step integration.

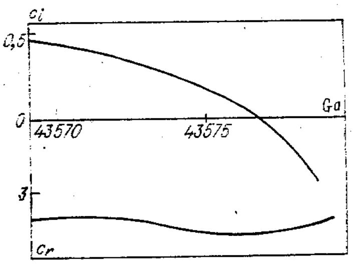

Fig. 3

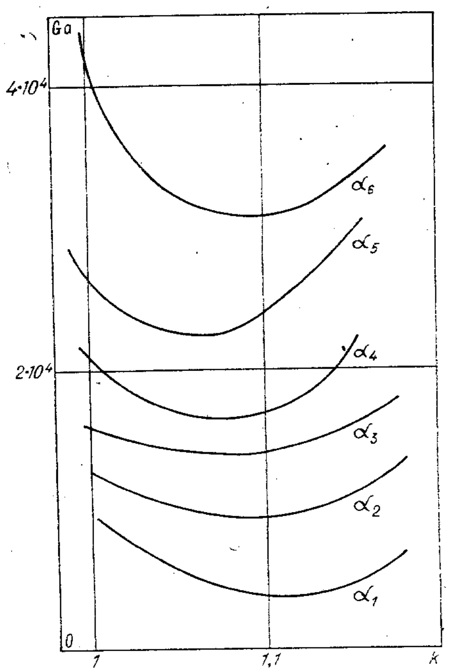

Fig. 4

3. Calculations performed for a wide range of values of the parameter α $(1 \leqq \alpha \leqq 50)$ show that instability of steady-state motion of the liquid with heavy particles is caused by the interaction of oppositely directed flows: the descending central flow and two ascending flows near the walls. Instability in the motion is produced by lower modes of hydrodynamic perturbations, while the decrements of normal perturbations prove to be complex (traveling perturbations). Figure 3 shows the decement c_i and phase velocity of perturbations as functions of the Galileo number ($\alpha = 50$, $k = 1$, $\tau_V = 0.92 \cdot 10^{-2}$).

The settling particles generate oscillatory (traveling) perturbations and encourage their transport. With decrease in the parameter α the stability of the flow induced by particle settling decreases. In fact, at low α (see Fig. 1) the particle distribution in the layer has a sharply expressed "tonguelike" character and the flow intensity is high (see Fig. 2); decrease in α leads to an increase in flow velocity and disruption of stability. This conclusion is confirmed by Fig. 4, which shows neutral stability curves ($c_i = 0$, $\tau_V = 0.92 \cdot 10^{-2}$, $\alpha_1 = 21$, $\alpha_2 = 31$, $\alpha_3 = 37$, $\alpha_4 = 40$, $\alpha_5 = 45$, $\alpha_6 = 50$). The character of the heavy particle distribution across the layer affects the stability of the flow induced by the impurity intensely.

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